Analysis of the Constrained Tendon Routing

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This appendix is for the paper "Exo-Glove Pinch: A Soft, Hand-Wearable Robot Designed Through Constrained Tendon Routing Analysis" submitted to RAL. Here, we present an analysis to help understanding the actuation characteristics of constrained tendon routing to use them when developing the robot. It starts by defining the tendon Jacobian [1], [2], which represents the relationship between the joint and the actuator position as

$$\dot{\mathbf{l}} = \mathbf{J_j}\dot{\mathbf{q}} + \mathbf{R}\dot{\theta} \tag{1}$$

where, $\mathbf{l} \in \mathbb{R}^{n\times 1}$, $\mathbf{J_j} \in \mathbb{R}^{n\times N}$, $\mathbf{q} \in \mathbb{R}^{N\times 1}$, $\mathbf{R} \in \mathbb{R}^{n\times m}$, and $\theta \in \mathbb{R}^{m\times 1}$ are the tendon length, tendon jacobian, joint angle, radius of the motor spool, and motor displacement, respectively. n, N, and m are the number of tendons, the number of joints, and the number of motors, respectively. We assume that m is smaller than N, as this paper focuses on methods for reducing the number of motors.

A. Analysis on position-constrained tendon routing (PTR)

PTR moves multiple joints at a fixed displacement ratio by applying position constraints. Assuming the actuation tendon is sufficiently stiff ($\dot{\mathbf{l}}$ is zero), the infinitesimal change in joint angle ($d\mathbf{q}$) for PTR can be derived from Eq(1) as

$$\begin{aligned} d\mathbf{q} &= -\mathbf{J_j}^{-1}\mathbf{R}d\theta \\ &= -\begin{bmatrix} r_{1,1}(q) & r_{1,2}(q) \\ r_{2,1}(q) & r_{2,2}(q) \end{bmatrix}^{-1} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} d\theta \end{aligned} \tag{2}$$

where, R_j is the radius of spool that pulls j-th tendon and $r_{i,j}$ is effective moment arm of the j-th tendon at i-th joint. $d\theta$ is the infinitesimal change in motor displacement. The equation below, which is expressed in scalar terms, is described to help the reader understand-i.e., this case shows when the system has two tendons. Note that we can only obtain the relationship between the infinitesimal change in joint angle and that of motor displacement because $r_{i,j}$ is the function of joint angle in suspended tendon routing in our case.

In PTR, tendon tension can not be obtained from the motor torque. We can only deduce that the sum of the pulley radii multiplied by the tension is equal to the motor torque as

$$\tau_{\mathbf{M}} = \sum_{i=1}^{cp} R_i T_i \tag{3}$$

where T_i is tension of the i-th tendon, and cp means the number of position constraints. However, this routing offers an advantage in controlling joint angles, because the kinematic relationship is strictly defined by Eq(2).

In applications that interact with the external environment, strictly defined position constraints may be undesirable. If one link is blocked by contact with objects, the system cannot move other links due to the position constraints between the tendons. In other words, the motion becomes not *adaptable* to the environment; the terminology *adaptability* will be described in detail later in subsection -C.1.

Researchers have tried to alleviate the position constraints to generate adaptive motions. This can be achieved by attaching elastic components in series with the tendons, making $\dot{\mathbf{l}}$ in Eq(1) non-zero. We refer to this method as compliant PTR in this paper. As the elastic components deform in proportion to the tension, the kinematic relationship between constrained joints changes as the tension increases. The joint configuration is defined as

$$\begin{aligned} d\mathbf{q} &= \mathbf{J_j}^{-1}(d\mathbf{l} - \mathbf{R}d\theta) \\ &= \begin{bmatrix} r_{1,1} & r_{1,2} \\ r_{2,1} & r_{2,2} \end{bmatrix}^{-1} \left(\begin{bmatrix} k_{s,1} & 0 \\ 0 & k_{s,2} \end{bmatrix}^{-1} \begin{bmatrix} T_{1,q} \\ T_{2,q} \end{bmatrix} - \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} d\theta \right) \end{aligned} \tag{4}$$

where, $T_{i,q}$ is the tension of the i-th tendon (when the joint angle is q) and $k_{s,i}$ is the stiffness of the elastic component attached at i-th tendon. The definitions of other variables remain the same as in Eq(2). Similar to Eq(2), the scalar term expression is used to show the two tendon case as an example. Due to the compliance of serially connected elastic components, the joint angle is affected by tension, allowing the robot system to adapt to the external environment.

B. Analysis on force-constrained tendon routing (FTR)

Another method to actuate multiple joints is applying force constraints to them. This routing pulls serially connected joints with a tendon, thereby constraining the tension applied across the joints. Since the tension remains consistent along the tendon, the same force is distributed across these joints.

A unique characteristic of FTR is that the joint angle cannot be determined solely by the tendon length. Instead, the joint angle is defined by analyzing the torque equilibrium equation because the force applied to these joints is constrained here. Using the virtual work principle from Eq(1), the joint torque applied by the tendon can be expressed as

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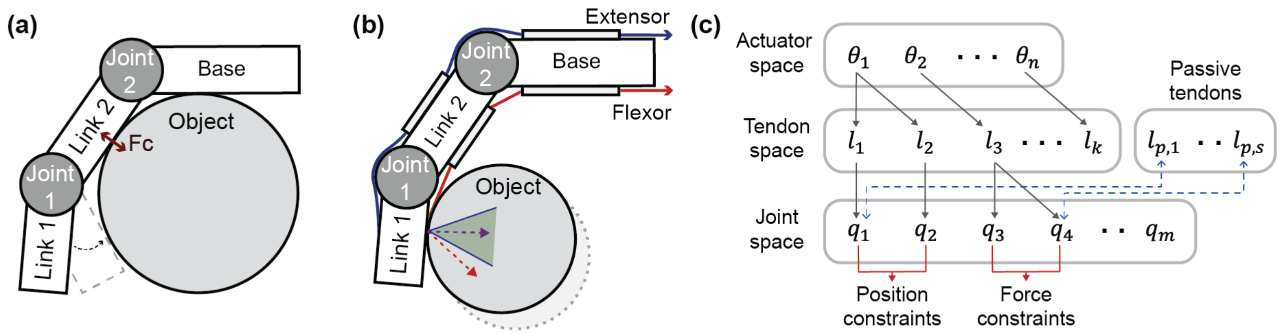

Fig. 1. Addressing adaptability and force capability issues in robots with fewer actuators. **(a)** Position constraints at the joints lead to adaptability issue—when Link2 is blocked, Link1 cannot move under this constraint type; force constraints should be used to solve this issue as will be explained in this appendix. **(b)** Force constraints raise force capability issues —limited control over fingertip force direction may cause objects to slip from the hand. **(c)** These issues arise from the position or force constraints applied by the tendon, which is inherent in designs with fewer actuators.

$$\tau_{\mathbf{r}} = \mathbf{J_j^T T}. \tag{5}$$

Under quasi-static conditions [3], the torque equilibrium equation can be represented as

$$\tau_{\mathbf{r}} - \kappa \Delta \mathbf{q} + \tau_{\mathbf{c}} = 0 \tag{6}$$

where $\tau_{\mathbf{c}} \in \mathbb{R}^{n_c \times 1}$, $\kappa \in \mathbb{R}^{N \times N}$, $\Delta \mathbf{q} \in \mathbb{R}^{n \times 1}$, and n_c are torque applied to the joint by contact force, joint stiffness matrix, joint angle displacement, and number of contact points, respectively. Joint stiffness describes the torsional spring-like behavior of human joints, determined by the stiffness of structures such as ligaments, tendons, muscles, and articular cartilage [4].

By combining Eqs(5) and (6), the joint angle $\mathbf{q}$ can be represented as

$$\begin{aligned} \mathbf{q} &= \kappa^{-1}(\tau_{\mathbf{r}} + \tau_{\mathbf{c}}) + \mathbf{q_0} \\ &= \kappa^{-1}(\mathbf{J_j^T T} + \tau_{\mathbf{c}}) + \mathbf{q_0} \end{aligned} \tag{7}$$

where $\mathbf{q_0}$ is the initial joint angle.

An interesting aspect in Eq(7) is that the joint angle $\mathbf{q}$ is influenced not only by tendon tension but also by the contact torque ($\tau_{\mathbf{c}}$). This term leads to *adaptive motions* which is one of the key advantages of FTRs. However, it is also important to consider *force capability* as these robots are not fully controllable. Detailed analyses on adaptability and force capability are provided in the following sections.

C. Motion characteristics of constrained tendon routings

1) Adaptability: Given that the number of actuators is fewer than the number of joints, the tendon Jacobian in Eq(1) has a null space. Consequently, the infinitesimal change in joint angle (d$\mathbf{q}$) is defined as

$$d\mathbf{q} = \mathbf{J_j}^{\dagger} \mathbf{R} d\theta + \alpha N(\mathbf{J_j}) \tag{8}$$

where $N(\mathbf{A})$ and $\mathbf{A}^{\dagger}$ represent null space and pseudo-inverse of the matrix $\mathbf{A}$, respectively. α is arbitrary real number that represents *span* of the null-space and it is determined by the contact torque applied by the external environment [1].

Therefore, the postures of robots using FTR are not fully defined by motor positions. While this may poses challenges in certain scenarios, researchers have found clever ways to leverage the existence of null space. This null space enables a unique motion pattern known as *adaptable motion* when interacting with the external environment. For instance, joints can continue to move even if other joints are blocked thanks to the null space of the jacobian (Fig. 1b). This adaptability is particularly beneficial in robotic grasping, where it helps increase the number of contact points between the robot and the object, leading to a more stable grasp with force closure.

When making adaptive motions in the situation shown in Fig. 1b, it is preferable to move link 1 (the link not in contact with the object) without increasing the contact force (F_c) at link 2 (the link in contact with the object). If the contact force increases prematurely, it could shift the object before achieving a stable grasp with force closure. Therefore, for stability, it is important to avoid increasing the contact torque (τ_c) at link 2. However, in practice, the tendon friction often increases the contact torque, so researchers have focused on minimizing the friction for adaptability. The infinitesimal change in contact torque when link 1 moves by dq_1 can be derived from Eq(5) and Eq(6) and is represented as

$$d\tau_{\mathbf{c}} = \begin{bmatrix} 0 & \frac{r_2}{r_1}\kappa_1 dq_1 \end{bmatrix}^T \tag{9}$$

where r_1 and r_2 are the tendon moment arms at joint 1 and 2, used to define tendon jacobian.

The human-robot system using PTR is not adaptable to the external environment because it eliminates the null space of the tendon Jacobian–meaning all the joint angles are strictly determined by motor displacement. However, with additional elastic components, the system can achieve adaptive motions, referred to as *soft synergy* [5]. Its adaptability can be figured out by deriving the contact torque, similar to Eq(9), using Eq(4) - (6). The infinitesimal change in the contact torque can be derived as

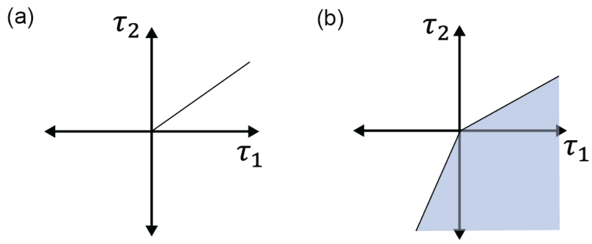

Fig. 2. Achievable torque manifolds in joint torque space. **(a)** shows the mechanically achievable torque manifold when using a single flexor, while **(b)** shows the torque manifold achievable with both a flexor and an extensor.

$$\begin{aligned} d\tau_c &\simeq J_j^T dT + \kappa dq \\ &= \begin{bmatrix} r_{1,1} & 0 \\ r_{1,2} & r_{2,2} \end{bmatrix} \begin{bmatrix} k_{s,1}dl_1 & k_{s,2}dl_2 \end{bmatrix}^T + \kappa dq \\ &= \begin{bmatrix} 0 & \frac{r_{1,2}}{r_{1,1}}\kappa_1 dq_1 + r_{2,2}k_{s,2}R_2 d\theta \end{bmatrix}^T \end{aligned} \tag{10}$$

assuming that the contact torque at joint 1 should be zero ($r_{1,1}k_{s,1}dl_1 + \kappa_1 dq_1 = 0$), as link 1 does not contact the object.

The term '$r_{2,2}k_{s,2}R_2 d\theta$', related to stiffness of the serially connected elastic component, makes difference between Eq(9) with Eq(10). If the stiffness is infinite (i.e. if the elastic component is absent), the contact torque will be diverse to the infinite, making the system lose adaptability. Conversely, if the stiffness of the serially connected elastic component approaches zero, two equations become the same. Therefore, using an elastic component with low stiffness may be preferred for adaptability. However, minimizing the stiffness can negatively impact the actuation bandwidth and control performance by reducing the actuation stiffness. Accordingly, researchers often prefer FTR instead of using additional elastic components when making adaptive motions.

The robot using FTR, however, may suffer difficulties in estimating joint position when there is significant friction. Since joint position is determined by force equilibrium, external disturbances, such as friction, can hinder the control of the joint position. PTR, on the other hand, eliminates the null space of the tendon Jacobian by increasing the number of rows (through the use of additional tendons) as can be seen in Eq(2). While eliminating the null space does not enable independent control of each joint, it is beneficial in terms of clearly defining the kinematic relationship between actuator position and joint angle.

2) Force capability: Adaptability in robots can be thought of as their ability to adjust the motions in response to external environmental changes. However, this responsiveness does not always produce the desired outcome. For instance, increasing tendon tension may not stabilize a grasp by either creating adaptive motion or increasing contact force. Instead, the increased tension could cause the object to slip out of the hand (Fig. 1c). The capacity to generate sufficient contact force without slipping is known as *force capability* [6].

To increase the contact force without slipping, it's important to consider the direction of the contact force. The force must be directed within the friction cone to prevent sliding on the object. However, in systems with fewer actuators, controlling the force direction becomes challenging due to the limited controllability. For instance, when using a tendon to actuate two joints, the joint torques can be expressed as

$$\begin{aligned} \tau_r &= J_j^T T \\ &= \begin{bmatrix} r_1 T & r_2 T \end{bmatrix}^T. \end{aligned} \tag{11}$$

Consequently, it may not always be possible to generate the exact joint torque required to direct the contact force within the friction cone.

Force capability can also be understood through a mechanically realizable manifold, representing a set of joint torques the robot can generate in torque space. When only a single flexor is used, as inferred from Eq(11), the manifold is expressed as a straight line in the joint torque space (Fig. 2a). If the joint torque, to keep the contact force within the friction cone, does not lie on this realizable torque manifold (straight line in this case), the grasp may become unstable.

The issue of insufficient force capability may be addressed by optimizing the design parameters of the robot [7]. However, this approach is not suitable in tendon-driven soft hand-wearable robot (TSHR) for the following reasons: 1) estimating appropriate contact points—key parameters in optimization—is challenging as the user, not the robot, controls the hand's movement toward the object; 2) even if the design parameters are optimized under certain assumptions, the results may not be effective in wearable robots since joint stiffness fluctuates with wrist position, which the robot cannot control. As illustrated in Fig. 2, adding just a single tendon significantly expands the area of achievable torque manifolds. Based on this insight, we developed a new version of the Exo-Glove, incorporating two tendons to have an achievable torque manifold similar to Fig. 2b.

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